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# Design and Testing of a Linear Compliant Mechanism with Adjustable Force Output 

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William Niemeier

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#### Abstract

This thesis presents a novel compliant mechanism with adjustable force output. The force comes from the bending of a rectangular cross section beam within the mechanism. By rotating this beam with a stepper motor, the force output of the mechanism changes. A model was made to simulate this mechanism, and a prototype was made based off of this data. A test apparatus was constructed around this mechanism, and a series of tests were performed. These tests adjusted parameters such as beam rotation speed and weight in order to characterize the system. Adjustments were made based on this information and the mechanism was refined. The results suggest the following. The speed has a negligible effect on the behavior of the system, while the weight, length of top link $r_{3}$, and position of bottom stop have a significant effect. Also, there is a large, consistent amount of hysteresis in the system. This is likely caused by the beam storing torsion or friction from the slider.


## CHAPTER 1: INTRODUCTION

### 1.1 Objective

The objective of this thesis is to describe the design a compliant low-stiffness mechanism with dynamically adjustable force levels that could be used as a suspension (vibration isolation) system, the design of a test apparatus, and the results of preliminary (static) tests of the mechanism's effectiveness. This novel mechanism is based on Howell's single-degree-offreedom constant-force mechanism (Howell 2001) but has been modified (Hasara 2018) to have two degrees of freedom. The first degree of freedom (the same as Howell's) allows for a large linear range of motion and passively exerts a low-stiffness, near constant force. The second degree of freedom (Hasara's modification) allows this force level to be changed.

### 1.2 Motivation

There are several applications for a mechanism of this nature, including in vehicle suspensions (Spelta, et. al. 2010) and minimizing damage from earthquakes (Araki, Asai, \& Masui 2009). This suspension reacts dynamically and by storing and releasing the vibrational energy it receives, may prevent the transmission of potentially damaging oscillations and shocks. This could be significant in earthquake-prone regions, on watercraft, and noisy environments like helicopters.

### 1.3 Scope

The scope of this thesis is to present the design and prototype of a compliant mechanism that is low stiffness, has adjustable force levels, and has properties that are well suited for use in vibration isolation, the design and prototyping of a test apparatus, and the preliminary (static) test
results. The suitability of the mechanism as a vibration isolator is addressed through the design process and by adjusting various parameters during testing.

### 1.4 Overview

The outline of this thesis is as follows. Chapter 2 is the background. It covers the relevant theory from compliant mechanisms, including Pseudo-Rigid-Body Models (PRBMs), Howell's original constant-force mechanism, Hasara's modifications, and the numerical model of this mechanism. Chapter 3 discusses the theory that went into designing the mechanism, and describes the mechanism that was built. Chapter 4 describes the entire test apparatus of which the mechanism is a part. Chapter 5 addresses the testing process and procedures. Chapter 6 presents the results of the testing. Chapter 7 provides discussion on the results and gives concluding remarks. Chapter 8 lists possible next steps that should follow the work presented here. Chapter 9 summarizes the contributions made in this thesis. Finally, there are 3 Appendices. Appendix A contains technical drawings of the different mechanical components, Appendix B has additional tables made from results data, and Appendix C lists the code for the Arduino programs that were used.

## CHAPTER 2: BACKGROUND

The background information for this thesis is in the area of compliant mechanisms; including Pseudo-Rigid-Body-Models (PRBMs), Howell's constant-force mechanism, and Hasara's modifications.

### 2.1 Compliant Mechanism Theory

Like rigid-link mechanisms, a compliant mechanism transfers or transforms motion, force, or energy. However, compliant mechanisms gain at least some of their mobility from the deflection of flexible members rather than from movable joints only (Howell 2001). Compliant mechanisms have many design benefits, which can be grouped into two main categories: increased performance (increased precision, increased reliability, reduced wear, reduced weight, reduced maintenance) and reduced cost (part-count reduction, reduced assembly time, simplified manufacturing processes) (Howell 2001).

Even though they offer many advantages, compliant mechanisms also have their share of challenges. One of the biggest challenges is the difficulty of analyzing and designing the mechanism (Howell 2001). By nature, the motion of compliant mechanisms involves large deflections of flexible beams. This means that the linearized beam equations that assume small deflections cannot be used. Other methods of analysis must be resorted to. These include: trial and error, large deflection analysis, and the Pseudo-Rigid-Body Model.

The Pseudo-Rigid-Body Model (PRBM) technique is used to predict the deflection path and force-deflection relationships of a flexible beam (Howell 2001). Compared to largedeflection analysis, it is both simpler and more efficient. It works by modeling a flexible beam as
two rigid links pinned together. At the pin joint, there is a torsional spring which represents the beam's resistance to deflection. An illustration of a flexible beam and its corresponding PRBM can be seen in Figure 2.1 and Figure 2.2 respectively.


Figure 2.1 Model of a deflected beam, adapted from (Howell 2001).


Figure 2.2 Pseudo-rigid body model of the beam in Figure 2.1, adapted from (Howell 2001).

One of the main drawbacks of the PRBM is that since it is an approximation, there is a known error. Also, it is only applicable to a given set of conditions, such as the fixed-pinned beam shown in the figures above. However, these conditions are general enough for the purposes of this thesis. For this reason, the PRBM method is used in this thesis.

### 2.2 Constant Force Mechanisms

An important property of compliant mechanisms is their ability to store energy. This can be harnessed to create mechanisms with special purposes, like the constant-force mechanism. Constant-force mechanisms produce a reaction force at the output port that does not change for a
large range of input motion (Howell 2001). Rigid-link versions of these mechanisms also exist, but their compliant counterparts have the advantage of requiring fewer parts.

The idea behind this mechanism is that as the vertical displacement $\Delta y$ gets larger, the mechanical advantage increases and balances the increasing moment from the torsional spring. In order for this to work properly, the ratio

$$
R=\frac{r_{3}}{r_{2}}
$$

must be an acceptable value. For the constant force mechanism shown in Figure 2.3, $\mathrm{R}=0.8853$.

### 2.3 Previous Design

Much of the design for the mechanism presented in this thesis comes from a loadadjustable compliant constant-force mechanism designed by Steven Hasara (Hasara 2018). This mechanism is a crank-slider, and features a compliant beam that can be rotated. The beam is framed by two sets of rigid links, one set on either side. The lengths of these links match those of the PRBM of the mechanism. This allows the path of the rigid links to match the natural path of the deflecting beam. The particularly clever part of this mechanism is that the compliant beam can rotate. The beam has a rectangular (1:2 aspect ratio) cross-section, and thus has a different stiffness depending on how the beam is rotated when it is deflected. When implemented in the mechanism, the constant-force value of the mechanism changes with the beam rotation. This design offers a theoretical 4 x increase of force output from $90^{\circ}$ beam rotation (Hasara 2018). The beam was rotated through the use of a NEMA 17 stepper motor with a 99.05:1 planetary gearbox. Its constant-force values were tested at various rotations using a tensile test machine.

The biggest differences between Hasara's mechanism and the one described in this thesis are as follows. First, the new mechanism is designed so that it can be attached to the crosshead of a tensile test machine and vertically oscillated. Second, a distance sensor and microcontroller
have been added. The controller gets data from the sensor and outputs to the stepper motor that rotates the compliant beam. Thirdly, the rigid-link lengths used in testing are different than Hasara's mechanism, and they are designed to be adjustable. The motor on this mechanism is larger, a NEMA 34, and has no gearbox attached to it. Also, the new mechanism is much more sturdy and robust, made of mostly aluminum instead of wood.

### 2.4 Numerical Model

The mechanism discussed in this thesis is a modification of a compliant constant force mechanism. The skeleton diagram of the mechanism and its equivalent PRBM are shown below in Figure 2.3.


Figure 2.3 Simplified diagram of mechanism (left); PRBM of simplified mechanism (right).

A numerical model was made of the pseudo rigid body model (PRBM). It initially calculated force, $F$, from slider displacement, $\Delta y$. Given $\Delta y, r_{2}, r_{3}$, and $\gamma$ (characteristic radius factor), the following was calculated:

$$
r_{1}=r_{2}+r_{3}-\Delta y, \quad K_{\theta}=\pi \gamma
$$

where $K_{\Theta}$ is the stiffness coefficient. The second moment of area, $I$, is needed to calculate the stiffness of the torsional spring $k_{l}$, but this is more complicated than usual. The beam, which has a rectangular cross section, is being bent at different rotations. This means the second moment of area is different at each beam angle $\Gamma$ and is calculated as follows:

$$
I_{z z}=\frac{b h^{3}}{12}, I_{y y}=\frac{b^{3} h}{12}, \quad I=\frac{I_{z z}+I_{y y}}{2}-\frac{I_{y y}-I_{z z}}{2} \cos 2 \Gamma
$$

where $b$ is the base of beam cross section, and $h$ is the height of beam cross section. This method is part of the 3D PRBM for Rectangular Cantilever Beams developed by Jairo Chimento (Chimento 2014). Next, the angular positions of the links ( $\theta_{2}$ and $\theta_{3}$ ) and $k_{1}$ are calculated:

$$
\theta_{2}=\cos ^{-1}\left(\frac{r_{1}^{2}+r_{2}^{2}-r_{3}^{2}}{2 r_{1} r_{2}}\right), \quad \theta_{3}=\sin ^{-1}\left(\frac{-r_{2} \sin \theta_{2}}{r_{3}}\right), \quad k_{1}=\frac{\gamma K_{\Theta} E I}{L}
$$

where $E$ is the elastic modulus and $L$ is the length of the original compliant link $\left(L=r_{2} / \gamma\right)$.
Finally, the force $F$ at a given $\Delta y$ is calculated using the following equation:

$$
F=\frac{r_{3} \cos \left(\theta_{3}\right) k_{1} \theta_{2}}{r_{2} r_{3} \sin \left(\theta_{2}-\theta_{3}\right)}
$$

This process is repeated at increasing displacements up to $40 \%$ of the total length, and all data is stored in Excel. The result is graphed as force over displacement, as is shown in Figure 2.4.

Here, $L=0.3 \mathrm{~m}, \gamma=0.83, r_{2}=0.249 \mathrm{~m}, r_{3}=0.220 \mathrm{~m}, E=1.7 \mathrm{GPa}, b=0.015 \mathrm{~m}$, and $h=0.0075 \mathrm{~m}$. Each line represents a different beam rotation ranging from $\Gamma=0^{\circ}$ to $\Gamma=90^{\circ}$.


Figure 2.4 Plot from the numerical model. Each line represents a different beam rotation. $0^{\circ}$ is the lowest, and $90^{\circ}$ is the highest. The maximum displacement shown is $40 \%$ of the total length. $\mathrm{R}=$ 0.8853

Using this numerical model, design parameters like the length ratio, $R$, can be adjusted. The length ratio was defined above as the ratio between $r_{3}$ and $r_{2}$, and for a constant force mechanism it equals 0.8853 . The plot in Figure 2.4 was made using this value, and it shows the horizontal lines that are to be expected from a constant-force mechanism. If $R$ is decreased, the lines curve downward, which means the system exhibits negative stiffness. While negative stiffness has been used successfully in vibration isolation, the negative stiffness element is
usually just part of a larger system which also contains positive stiffness elements (Le \& Ahn 2013) (Yang, Xiong, \& Xing 2013) (Shahadat et. al. 2015).

## CHAPTER 3: PROTOTYPE

Based on the theory just discussed, the prototype was designed. There are many components that went into the overall structure of the apparatus, but in this chapter the focus is narrowed to the crank-slider mechanism portion. This includes: the side links, the joints, the compliant beam, the slider, and their attachments to the test apparatus. The labeled CAD model of the mechanism is shown below in Figure 3.1. The rest of the test apparatus will be discussed in the next chapter. This chapter gives qualitative descriptions and the rationale for the design choices. The dimensioned drawings of each component are given in Appendix A.


Figure 3.1 The mechanism is shown bare (left) and with the relevant structure (right).

### 3.1 Links and Joints

Before ordering the links, a value for the length ratio, $R$, was chosen. Because the system discussed in this thesis does not have additional elements to compliment the negative stiffness
that comes from $R<0.8853$, negative stiffness was determined to be undesirable and values of $R$ $<0.8853$ were ignored. Increasing R makes the lines curve upward, resulting in a desirable increase in stiffness. However, there is an upper limit. If $R$ is increased too much, the mechanism gets too long with respect to the length of the beam, and the beam bends a lot to achieve the same displacement. In Table 3.1, three different $R$ values are compared. For each one, the bending angle of the beam $\left(\theta_{2}\right)$ is shown for 4 different displacements.

Table 3.1 Bending angle of beam, $\theta_{2}$, vs displacement, $\Delta y$, for different length ratios, $R$.

| Displacement | $\theta 2$ |  |  |
| :--- | :--- | :--- | :--- |
|  | $R=0.8853$ | $\mathrm{R}=1.2$ | $\mathrm{R}=2$ |
| $10 \%$ length | $25.0^{\circ}$ | $29.9^{\circ}$ | $38.8^{\circ}$ |
| $20 \%$ length | $36.3^{\circ}$ | $43.0^{\circ}$ | $57.2^{\circ}$ |
| $30 \%$ length | $44.2^{\circ}$ | $53.9^{\circ}$ | $73.7^{\circ}$ |
| $40 \%$ length | $51.3^{\circ}$ | $63.9^{\circ}$ | $91.0^{\circ}$ |

When the beam bends past $60^{\circ}$, excessive plastic deformation occurs. Also, the design does not require that the max displacement get above $40 \%$. Based on this data, I decided on a length ratio value of 1.2. This gives the highest stiffness without overbending the beam. The same plot from Figure 2.4 is shown below in Figure 3.2, but with $R$ changed to 1.2.


Figure 3.2 Plot from the numerical model. Each line represents a different beam rotation. $0^{\circ}$ is the lowest, and $90^{\circ}$ is the highest. The maximum displacement shown is $40 \%$ of the total length. $R=$ 1.2

The next challenge was finding a source for the four links (top left, top right, bottom left, and bottom right). The links were required to be stiff, light, strong, preferably adjustable, and have solid joints that could rotate freely with minimal friction. This last requirement was particularly constraining. The joint needed to be tightened without increasing friction. I chose to use an assembly of Actobotics 33-Hole Aluminum Beams and Face Thru-Hole Pillow Blocks. A total of ten pillow blocks were used, and were fastened to the links at either end, allowing free rotation because of their bearings. To make a joint, a screw was passed through selected holes in the two links and small washers were used to keep the links from touching. Then, a locknut was
fastened behind the link to hold the joint together. The holes in the links allowed for convenient adjustability in link length by moving the attachment position of the pillow blocks. A CAD model of a link with an attached pillow block is shown in Figure 3.3.


Figure 3.3 A rigid link (Actobotics 33-hole aluminum beam) with a face thru-hole pillow block on the end.

### 3.2 Compliant Beam

Next, a compliant beam was chosen. Two materials were initially considered: polypropylene and steel. Polypropylene (PP) beams were used for two reasons. The first reason was the ease of producing PP beams. They can be cut out of a sheet using USF's Compliant Mechanisms Research Lab (CMRL) laser cutter. Figure 3.4 shows one of the laser-cut PP beams. The second reason is that the Lab has a supply of PP sheets and they are relatively inexpensive. This allows for both mass production and easy adjustability. It became very practical to have a new beam for every test, and it was easy to cut a different beam if the current one is not desirable due to excessive plastic deformation. By contrast, the steel beams are expensive and difficult to machine to specific cross-sectional areas, making it impractical to obtain even one steel beam, much less multiple beams for testing purposes.

Figure 3.4 One of the polypropylene beams made using the laser cutter.

### 3.3 Beam Couplings

The bottom of the beam needed to be attached to the motor and the beam top attached to the top end of the bottom link. These attachments required fixtures that were custom made using 3D printing (Fused Deposition Modeling i.e. FDM) in USF's Advanced Visualization Center.

The bottom fixture was a simple coupling that joined the PP beam to the shaft of the stepper motor. Both the beam and the motor shaft are press fit into the coupling. A CAD model of the coupling is shown in Figure 3.5.


Figure 3.5 Two views of the beam-motor coupling. In the left image, the aperture that accepts the rectangular compliant beam is shown. In the right image, the aperture that accepts the motor shaft is shown.

The top fixture has two separate parts. The smaller part is press fit onto the top of the compliant beam and ends in a cylindrical boss that fits loosely into a bearing contained by the second, larger part. This larger part is joined on both the left and right sides to the joint between the top and bottom links, and can freely rotate about the joint's axis. This fixture helps constrain the bending of the beam. The free rotation constrains the position of the top of the beam, but not the angle, while the boss loosely fitting into the bearing lets the beam slip slightly so that it is never in compression. Both parts of this fixture are shown in Figure 3.6.


Figure 3.6 Assembled (top) and exploded (bottom) view of the fixture for the beam top.

### 3.4 Slider

The slider was part of the Futura T-slots system that most of the test structure is made out of. It is designed to slide along one of the aluminum structural members. The contact surface is made of ultra-high-molecular-weight polyethylene (UHMWPE). This material has a very low coefficient of friction, lower than nylon and comparable to PTFE (Teflon). However, it has better abrasion resistance than PTFE. On top of that, it is self-lubricating. These properties make it
ideal for use as a slider in this mechanism, and it can be seen in Figure 3.7. In order to connect it to the links, two 3 " aluminum channels with pillow blocks were fastened to the slider. The links were then fastened to the pillow blocks.


Figure 3.7 A CAD model of the slider.

## CHAPTER 4: TEST APPARATUS

Now that the prototype's components have been discussed, this chapter describes the test apparatus. The test apparatus components are both mechanical and electrical.

### 4.1 Mechanical Components

Perhaps one of the biggest early decisions was choosing the stepper motor. The shaft of the motor is coupled to the beam, allowing the motor to control the beam's twist angle. Based on the numerical model that was made of the system in Chapter 2, it was predicted that the motor may have to withstand torques of up to 10-12 Nm. Because of this, I chose the Nema 34 Hybrid Stepper Motor which has a holding torque of 13 Nm . This conservative choice gives more than enough power to control the beam and deal with any loads it may endure.

Another important choice was the distance sensors. We needed a distance sensor that was accurate, fast, had a range of about two feet, was compatible with the Arduino microprocessor, was relatively small, and was reasonably priced. The Migatron LCU-40APW was chosen. It is an analog ultrasonic sensor with a range of $4 "-40 "$, takes about 100 measurements per second, and has a resolution of 0.039 ". It also was relatively low cost, and fits comfortably in the palm of your hand. The resolution mentioned is calculated with respect to the system discussed in this thesis, which uses an Arduino to measure the analog voltage output of the sensor. It is sufficient for this thesis, but a finer resolution may be possible with another system.

As mentioned briefly in the previous chapter, most of the structure of the apparatus was made using parts from the Futura T-slots system. Its main feature is precision aluminum extrusions which can be cut and joined together to make strong, stable structures. All of the
supporting framework in this test apparatus is made of extrusions with a cross section that measures 1.5 " square. They are fastened to each other along their slots, which is important because it means that the joints can be loosened and the components can slide to a new location if desired. This makes design and assembly less difficult, and also allows for easy modifications to the apparatus. These were both desirable qualities. The T-slot frame supports the bottom links of the mechanism, the slider at the top, the sensor fixture, and the stepper motor.

The mechanism's range of motion needed to be limited for a few reasons. First is that with the current design, the slider will fall off of its extrusion if it goes too low, and the mechanism will hit a dead center if it reaches the upper limit of its range. From the dead center, the mechanism could bend "elbow in" instead of "elbow out". This will cause the top beam fixture to interfere with the structure, and is not the intended use of the mechanism. But the main reason to limit the range is to avoid overstressing the beam. The more the beam is allowed to bend, the quicker it will fatigue. To reduce the motion range, two hard stops were placed on the slider extrusion. Each hard stop is a large nut that is bolted to the slot, blocking the motion of the slider. The top stop was placed just below the upper limit of the mechanism, not allowing it to fully straighten out. The bottom stop was placed about halfway down initially, but was tested at different positions. Because of the T-slots, the hard stops could be securely fastened anywhere along the length of the extrusion.

The complete test apparatus (electrical components not pictured) is shown in Figure 4.1.


Figure 4.1 Completed test apparatus.

### 4.2 Electrical Components

The electrical components of the system are gathered together for organization in a lasercut wooden case. The electrical components have two functions: to control the stepper motor, and to measure displacements.

The two biggest parts are the power supply and the stepper motor driver. The power supply receives 120 V AC from a standard outlet and outputs 48 V DC at 7.3 A . This powers the driver and in turn, the stepper motor. The driver is an MA860H Microstep Driver, which is shown in Figure 4.2. It has 8 inputs and 4 outputs. Two inputs receive power from the power supply and the remaining 6 come from the Arduino and are used to control the motor. They are
split into 3 +/- pairs, labeled "PULS", "DIR", and "ENA". The "+" is the actual input, and the "$"$ is its ground. Each input is either low or high. ENA stands for Enable, and controls the power state of the driver. As long as this is low, the driver is powered on. DIR stands for direction, and controls the direction the motor steps in. A low signal sets the direction to clockwise, while a high signal is counterclockwise. Finally, PULS stands for pulse, and providing an electrical pulse (low to high) advances the motor one step in the chosen direction. The 4 outputs on the driver correspond to the 4 wires from the stepper motor. They are labeled A+, A-, B+, B-. They match with the motor's colored wires as follows: $\mathrm{A}+=$ red, $\mathrm{A}-=$ green, $\mathrm{B}+=$ yellow, $\mathrm{B}-=$ blue. The wires that run from the power supply and from the stepper motor to the driver are 18 gauge. This is thicker than the rest of the wiring, which is either 28 or 24 gauge. The driver has several different resolution options for the stepper motor, from 400 to 51200 pulses/revolution. I chose the lowest option, 400 pulse/rev, because it was the fastest and having $0.9^{\circ}$ per step was fine enough.


Figure 4.2 MA860H Microstep Driver

The other main electronic component is the microcontroller. This had to serve as the heart of the electronic system, being able to interact with all the components. Although there were other worthy candidates, the Arduino UNO was chosen mostly because of my own familiarity with it. However, its popularity means that there is a lot of online support which was very helpful during the coding process.

One of the early electrical challenges was limiting the voltage output of the distance sensors. They output 1-10V, which corresponded linearly with their 4-40" range. However, the Arduino can only handle 5 V maximum. To fix this, I made a voltage divider circuit, which scales
the voltage by a factor of 0.5 . This made the new voltage range $0.5-5 \mathrm{~V}$, which was perfect for the system. The diagram of the voltage divider is shown in Figure 4.3.


Figure 4.3 Diagram of the voltage divider circuit used in the system. It scales the output voltage of the sensors by a factor of 0.5 .

All of the electronic components were mounted in a wooden case, both for ease of transport and to protect them. The case was custom built from laser-cut wood, and has 6 ports: one each for the two distance sensors, and one for each of the four stepper motor wires. The finished electronics assembly is shown in Figure 4.4 and the corresponding wiring diagram can be seen in Figure 4.5.


Figure 4.4 The electrical components in the laser-cut case. The left image shows the front of the case, while the right image shows the ports on the back.


Figure 4.5 Wiring diagram for all electronic components.
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## CHAPTER 5: TEST PROCEDURES

Ten types of tests were performed on the system. They were all based on the same Arduino code, but with small variations. The basic process of the original code was as follows. The motor twists the base of the beam from $0^{\circ}$ and $108^{\circ}$ and back, three times, while collecting distance data from the sensor. During the actual test each program ran 3 times back to back, so that the system completed 9 loops total. The various tests performed used this template but changed some of the test parameters, including: the position of bottom hard stop, the speed of beam rotation, length of top link $\left(r_{3}\right)$, the weight of top platform, and the twist reversal angle. Before each test, the zero position of the beam is set manually using a special program.

The parameters of Test 1 will be described in detail, and the other tests are described in less detail, assuming that the non-specified features are identical with this first test. Starting at a twist angle, $\Gamma=1$, the motor steps up to 120 steps $\left(108^{\circ}\right)$ and back down to one at a speed of 0.978 steps/s. This loop is repeated 9 times. During each step, 10 readings are taken from the distance sensor and averaged, then this value is logged with the current step number. This process is repeated 10 times, resulting in total 100 measurements and 10 logged values for every step. The top link $r_{3}$ is $11.55 "$ long, while the bottom link $r_{2}$ is $9.625 "$. The bottom stop is 14.375 " above the bottom joint of $r_{3}$, which allows for the slider to move up to about $29 \%$ of the effective mechanism length. The top stop is 22.75 " above the same joint. This is about 1 " less than the maximum platform height at dead center. The polypropylene beam is $13.25^{\prime \prime}$ long with an effective length of $11.25^{\prime \prime}$ (it is inserted into 1 " deep slots at either end), and a cross section of $0.5 "$ by $0.25 "$. It is also new, meaning that it has not been stressed or used in any previous tests.

The data is collected and imported into Excel. The voltage value reported by the Arduino (01023) is converted into the distance $r_{1}$, which is measured in inches from the bottom joint of link $r_{2}$ to the top joint of link $r_{3}$. This is the conversion equation:

$$
r_{1}=25.4375-\left(\frac{V+5.4577}{25.433}\right)
$$

where $V$ is voltage.
In Test 2 two things were changed. The bottom stop was moved up 1.5 " to a position of $15.875^{\prime \prime}$. Also, the joint near the top of the beam where the cylindrical boss mates with the bearing was lubricated with WD40.

For Test 3, the bottom stop was moved up $1.5 "$ like in Test 2, and the bearing joint was lubricated. The top link $r_{3}$ was shortened by 0.77 " to a length of 10.78 ". The top stop was moved down 0.8 " to a height of 21.95 " to compensate for the shortening of the top link.

Test 4 had the bottom stop moved up 3" to a position of 17.375 ". The bearing joint was lubricated.

In Test 5, the bottom stop was moved up 3", the bearing joint was lubricated, and the speed was increased. A new Arduino code was used that decreased the delay between measurements from 100 ms to 50 ms . This increased the rotation speed to 1.919 steps $/ \mathrm{s}$.

In Test 6, the bottom stop was moved up 3", the bearing joint was lubricated, and the speed was increased again. A new Arduino code decreased the delay between measurements to 25 ms , which made the rotation speed 3.676 steps $/ \mathrm{s}$.

Test 7 was the final speed test. The bottom stop was moved up 3" and the bearing joint was lubricated. The new Arduino code set the delay between measurements to 10 ms , which made the rotation speed 6.803 steps/s.

Test 8 was the first hysteresis test. The bottom stop was moved up 3 " and the bearing joint was lubricated. A new Arduino code was used, and the delay between measurements was set back to 100 ms . However, instead of rotating between 1 and 120 steps, it rotates up to 80 , then down to 60 , then back up to 80. It rotates between 60 and 80 six times total, before descending from 80 back down to 1 . This code is only run once.

Test 9 was the second hysteresis test. It is the same as Test 8 , but the rotation range was changed. The bottom stop was moved up 3", the bearing joint was lubricated, and a new Arduino code was used. This time, the motor rotates up to 75 , then goes back and forth between 35 and 75 6 times. After, it goes from 75 back down to 1 .

Test 10 was the weight test. It is identical to Test 4 but with weight added. The bottom stop was moved up 3" and the bearing joint was lubricated. Two weight plates weighing 0.25 lbs each were fastened to the platform, for a total of 0.5 lbs of added weight.

## CHAPTER 6: RESULTS

Now that the test procedures have been described, the data collected from each test are presented. The first test, Test 1, consists of an initial run, Test 1.1, and its replicate, Test 1.2, which can be seen in Figures 6.1 and 6.2, respectively.


Figure 6.1 Plot of Test 1.1. R1 is plotted over $\Gamma$. The line color gets lighter with each loop. The arrows show the path of the mechanism.


Figure 6.2 Plot of Test 1.2. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.

Next, the results of Test 2 are shown in Figure 6.3. For this test, the bottom stop was moved up $1.5 "$. The test was performed only once.


Figure 6.3 Plot of Test $2 . R 1$ is plotted over $\Gamma$. The line color gets lighter with each loop.
For Test 3, the bottom stop was left unchanged from Test 2 but the top link was shortened. These results can be seen in Figure 6.4.


Figure 6.4 Plot of Test 3. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.

Test 4 had the bottom stop moved up 3". There was an initial run, and 2 replicates.
Results are shown in Figures 6.5, 6.6, and 6.7.


Figure 6.5 Plot of Test 4.1. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.


Figure 6.6 Plot of Test 4.2. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.


Figure 6.7 Plot of Test 4.3. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.

For Test 5, speed was increased to 1.919 steps/s. Results are shown in Figure 6.8.


Figure 6.8 Plot of Test $5 . R 1$ is plotted over $\Gamma$. The line color gets lighter with each loop.

For Test 6, speed was increased to 3.676 steps/s. Results are shown in Figure 6.9.


Figure 6.9 Plot of Test $6 . R 1$ is plotted over $\Gamma$. The line color gets lighter with each loop.

For Test 7, speed was increased to 6.803 steps/s. There was an initial run, and 2
replicates. Results are shown in Figures 6.10, 6.11, and 6.12.


Figure 6.10 Plot of Test 7.1. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.


Figure 6.11 Plot of Test 7.2. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.


Figure 6.12 Plot of Test 7.3. R1 is plotted over $\Gamma$. The line color gets lighter with each loop.

Test 8 rotated the motor back and forth in a 20 -step range from $60-80$. Figure 6.13 shows the results.


Figure 6.13 Plot of Test 8. R1 is plotted over $\Gamma$. The lines overlap in the top right corner.

Test 9 rotated the motor back and forth in a 40 -step range between 35 and 75 steps. The results are shown in Figure 6.14.


Figure 6.14 Plot of Test $9 . R 1$ is plotted over $\Gamma$.

Test 10 added 0.5 lbs of weight to the platform. Figure 6.15 shows the results.


Figure 6.15 Plot of Test $10 . R 1$ is plotted over $\Gamma$. The line color gets lighter with each loop.

## CHAPTER 7: DISCUSSION AND CONCLUSIONS

Now that the test data has been visualized, it will be analyzed and discussed. In the previous chapter, each test was accompanied with a plot which showed vertical position $\left(r_{1}\right)$ against motor position $(\Gamma)$ for a series of 9 loops. The majority of these loops had nearly identical maximum and minimum vertical positions. This range was taken and split into equal quartiles for each loop, and the corresponding motor position (steps) at each of those values was recorded. Since the tests showed separate but consistent behavior for the platform rising compared to falling, this process was applied to both directions separately. This is shown for Test 4.1 in Table 7.1. Then this data was visualized by using a dual series of boxplots. The upper row represents rising, and the lower row represents falling. This can be seen in Figure 7.1. The tables for every other test will not be shown here, but can be found in Appendix B.

In this chapter, the tests will be compared to quantify the effect of 5 different parameters: speed of motor rotation, length of the top link $r_{3}$, position of the bottom stop, weight of the top platform, and twist reversal angle.

Table 7.1 Min, max, midpoint, and quartile values with their corresponding motor positions for the initial run of Test 4.

Test 4.1

| Up |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.932 | 20.936 | 20.932 | 20.932 | 20.924 | 20.933 | 20.928 | 20.933 | 20.927 |
| Beam Angle | 85.1 | 90.3 | 93.6 | 95.9 | 99.3 | 101.2 | 101.3 | 103.8 | 105.9 |
| Min | 18.203 | 18.206 | 18.210 | 18.203 | 18.203 | 18.204 | 18.196 | 18.202 | 18.202 |
| Beam Angle | 66.3 | 68.1 | 69.2 | 70.2 | 69.4 | 69.5 | 69.5 | 69.6 | 69 |
| Midpoint | 19.568 | 19.571 | 19.571 | 19.567 | 19.564 | 19.569 | 19.562 | 19.568 | 19.565 |
| Beam Angle | 69.4 | 71.6 | 72.1 | 72.7 | 72.7 | 72.6 | 73 | 73.1 | 73.2 |
| 1/4 point | 18.885 | 18.888 | 18.890 | 18.885 | 18.884 | 18.887 | 18.879 | 18.885 | 18.884 |
| Beam Angle | 68.7 | 71.2 | 71.5 | 72.3 | 72.2 | 72.1 | 72.2 | 72.4 | 72.3 |
| 3/4 point | 20.250 | 20.253 | 20.252 | 20.249 | 20.244 | 20.251 | 20.245 | 20.250 | 20.246 |
| Beam Angle | 72.2 | 75 | 75.7 | 76.3 | 76.6 | 77.2 | 77.4 | 77.7 | 78.1 |
|  |  |  |  | Down |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.946 | 20.944 | 20.943 | 20.943 | 20.942 | 20.946 | 20.941 | 20.940 | 20.945 |
| Beam Angle | 47.5 | 54.2 | 57.4 | 58.7 | 60.4 | 60.9 | 62.3 | 63.5 | 64.4 |
| Min | 18.222 | 18.226 | 18.224 | 18.224 | 18.229 | 18.224 | 18.220 | 18.228 | 18.227 |
| Beam Angle | 34.8 | 36 | 36.7 | 37.9 | 38.5 | 39.2 | 39.9 | 40.5 | 40.4 |
| Midpoint | 19.584 | 19.585 | 19.583 | 19.584 | 19.585 | 19.585 | 19.581 | 19.584 | 19.586 |
| Beam Angle | 37 | 38.6 | 39.7 | 40.6 | 41.4 | 41.8 | 42.4 | 42.8 | 43.4 |
| 1/4 point | 18.903 | 18.905 | 18.903 | 18.904 | 18.907 | 18.904 | 18.900 | 18.906 | 18.906 |
| Beam Angle | 36.4 | 37.3 | 38.5 | 39 | 39.8 | 40.7 | 41.5 | 42.3 | 42.4 |
| 3/4 point | 20.265 | 20.265 | 20.263 | 20.264 | 20.263 | 20.265 | 20.261 | 20.262 | 20.266 |
| Beam Angle | 38.7 | 40.5 | 41.7 | 43 | 44.3 | 44.8 | 45.7 | 46.5 | 46.8 |
|  |  |  |  |  |  |  |  |  |  |

Figure 7.1 Boxplots of the initial run of Test 4. Each one represents the motor position at different values of $r_{l}(\min , 25 \%$, midpoint, $75 \%$, max). The top row describes the system moving up at each iteration, and the bottom row describes it moving down.

### 7.1 Motor Rotation Speed

Using this visualization, Test 4 was compared against Test 7. These were alike in procedure except for speed. Test 4 had the slowest speed ( 0.978 steps/s) while Test 7 had the fastest speed ( $6.803 \mathrm{steps} / \mathrm{s}$ ). First, all 3 runs of Test 4 were compared, as seen in Figure 7.2. The same was done in Figure 7.3 for the 3 runs of Test 7.


Figure 7.2 Boxplots comparing the initial run of Test 4 and its replicates. Each plot represents the motor position at different values of $r_{l}(\mathrm{~min}, 25 \%$, midpoint, $75 \%$, $\max$ ). The top row describes the system moving up at each iteration, and the bottom row describes it moving down. Each color represents a different run: Test $4.1=$ red, Test $4.2=$ green, Test $4.3=$ blue.


Figure 7.3 Boxplots comparing the initial run of Test 7 and its replicates. Each one represents the motor position at different values of $r_{l}(\min , 25 \%$, midpoint, $75 \%$, max). The top row describes the system moving up at each iteration, and the bottom row describes it moving down. Each color represents a different run: Test $7.1=$ red, Test $7.2=$ green, Test $7.3=$ blue .

First, it is seen that the top row of boxes in Test 4 are taller than in Test 7. This means that the amount of motor steps between the first and third quartiles is greater in Test 4 when the mechanism is rising. This distance between quartiles is comparable to the slope of the line during its rise or fall in the plots shown in Chapter 6. Having the slope near vertical is not ideal for a controllable system, so having short boxes is not ideal either. Looking at the bottom row, Test 7 is the one with the taller boxes. This is interesting, because the boxes in Test 7's bottom row are
also much taller than its top row. This indicates a difference in slope in Test 7 between ascending and descending. This adds complication to a controllable system and is not ideal.

Below in Figure 7.4, Tests 4 and 7 are compared on the same graph. It is a minimal version of the previous two plots, with just the midpoint values being displayed. The midpoint values give a good one-number summary of the position of the rise and fall curves. The plot shows clearly the difference in motor position between the two tests, and the difference between the rise and fall of each test.


Figure 7.4 Plot of the midpoints of all runs of Tests 4 and 7 at each loop iteration. Here, green represents Test 4 and blue is Test 7. Circle markers show the initial run, triangles show replicate 1 , and square markers show replicate 2.

The plot seems to show that Test 7 is lagging behind Test 4. It takes 2-4 degrees more for Test 7 to reach the same point as Test 4, on both rise and fall. This makes the effects of hysteresis more pronounced for Test 7. The amount of hysteresis can be roughly estimated by the size of the
gap between the top midpoint lines and bottom midpoint lines of the same color and style. But other factors should be taken into account before making any solid conclusions. The difference between the lines of Test 4 and those of Test 7 is comparable to differences within each test. Specifically, the difference between the first and last loop iteration of each run and the difference between runs.

Speed of motor rotation cannot be determined to significantly affect the motor position of the rise and fall curves. However, higher speeds may increase the difference in slope between rise and fall curves. This could lead to control problems in the future.

### 7.2 Position of Bottom Stop

Next, I look at how the position of the bottom stop affects the behavior of the system. Tests 1, 2, and 4 are compared to isolate this parameter. The difference between these three tests is only the position of the bottom stop. This can be seen in Figure 7.5, where one run is chosen from each test, and their respective plots from Chapter 6 are superimposed on each other. The minimum $r_{l}$ value for each color show where that test's bottom stop is.

Note that each color makes a similar shape, but that shape's dimensions vary between colors as the bottom stop moves upward. Not only does the shape get shorter, it gets skinnier and also shifts to the left. The height of the shape describes the vertical range of the mechanism. A large range is desirable, so shorter is not necessarily better. However, the shifting to the left is a good thing and may outweigh that. The shape's horizontal position indicates the active range of the motor rotation. It is desirable to have this motor range not extend past 90 degrees. This is because the beam should not have to rotate past $90^{\circ}$ to move the platform to its maximum height. The beam also fatigues more quickly the lower the bottom stop is. This is shown in the figure in two ways. First, the lines for each loop are more spaced out the lower the bottom stop is. Also, in
the tests that use the lower two positions (red and blue on the plot), the max height decreases with each loop. It makes sense that the beam fatigues more rapidly with a lower bottom stop because it is being bent further and therefore experiences greater stress.


Figure 7.5 Superimposed plots of Test 1.2 (blue), Test 2 (red), and Test 4.3 (green).

Now, the slopes of the lines during each test will be compared. The boxplots shown in Figures 7.6, 7.7, and 7.2 correspond to Tests 1, 2, and 4 respectively. Examining these plots shows a weak correlation between box height and bottom stop position. Interestingly, the middle position (Test 2) has the tallest boxes. However, the plot of Test 2 shown in Figure 7.5 has a unique bend in the lines about halfway up, and this may be affecting the boxplots. The midpoint lines in the top row of boxes are halfway up or higher, which is unusual because they are normally near the bottom of the box next to the $25 \%$ line. Also, Test 2 was only performed once so it may not be a good idea to make conclusions on this data. Something that the boxplots do
not account for is the vertical range, which affects the slope. Slope $=y / x$, and the box height only provides the x . The vertical range for Test 4 is about half that of Test 1 , so if we doubled the height of Test 4's boxes with respect to Test 1 , that would be a more accurate estimation of the slope. Since Test 4 and Test 1's boxes are similar in height as displayed, this doubling would easily show that the slopes of Test 4 is gentler and more desirable than those of Test 1.


Figure 7.6 Boxplots comparing all runs of Test 1. Each boxplot represents the motor position at different values of $r_{1}(\mathrm{~min}, 25 \%$, midpoint, $75 \%$, max). The top row describes the system moving up at each iteration, and the bottom row describes it moving down. Test 1.1 is red, and Test 1.2 is green.


Figure 7.7 Boxplots of Test 2. Each one represents the motor position at different values of $r_{1}$ (min, $25 \%$, midpoint, $75 \%$, max). The top row describes the system moving up at each iteration, and the bottom row describes it moving down.

Finally, the positions of the rise/fall curves are compared using a different plot. The midpoints from all three tests were plotted and can be seen in Figure 7.8. The vertical shifting between colors reflects the horizontal shifting in Figure 7.5. The other thing to notice is that the distance between rise/fall lines within colors slightly decreases as the bottom stop moves up.

This suggests that a higher bottom stop position reduces the effect of hysteresis.

Because of these reasons, the highest bottom stop position (as used in Test 4) is determined to be the most desirable. The range of motion is the lowest of the three tests but other
factors make up for it. The full range of motion is possible without the motor exceeding 100 steps, the beam fatigues the least quickly, the slopes of the rise/fall curves are the gentlest, and the effect of hysteresis is minimized.


Figure 7.8 Plot of the midpoints of all runs of Tests 1, 2, and 4 at each loop iteration. Here, Test 1 is blue, Test 2 is red, and Test 4 is green. Circle markers show the initial run, triangles show replicate 1 , and square markers show replicate 2 .

### 7.3 Length of Top Link

Now I will explore how the length of the top link, $r_{3}$, affects the system. This variable is isolated by comparing Test 2 and Test 3. In Test $2 r_{3}$ is $11.55 "$ long, and in Test 3 it gets reduced by 0.77 " to be 10.78 " long. This means the ratio $\mathrm{R}=1.2$ in Test 2 , while $\mathrm{R}=1.12$ in Test 3 . The plots of Tests 2 and 3 from Figures 6.3 and 6.4 are superimposed below in Figure 7.9. Shortening $r_{3}$ decreases the vertical range of the system, as seen in the figure.


Figure 7.9 Superimposed plots of Test 2 (red) and Test 3 (blue).

The slopes of the fall curves seem steeper in Test 2, while the rise curves have similar slopes in both tests. The difference between rise slopes and fall slopes is greater in Test 3, and is not ideal. This can be seen clearly in the boxplots of Test 3, shown below in Figure 7.10. The boxes in the top row are much shorter than the boxes in the bottom row.

Because of the decrease in range of motion and the increase in slope difference between rise and fall, Test 3 gave a less controllable system than Test 2 . For this reason, $r_{3}=11.55$ " and R $=1.2$ are determined to be superior.


Figure 7.10 Boxplots of Test 3. Each one represents the motor position at different values of $r_{1}$ ( $\mathrm{min}, 25 \%$, midpoint, $75 \%$, max). The top row describes the system moving up at each iteration, and the bottom row describes it moving down.

### 7.4 Weight Added to Platform

Now I will analyze the effect of added weight on the system. Test 4 and Test 10 will be compared to isolate the weight parameter. The only difference between these tests is that Test 10 has 0.5 lbs . added to the top platform, while Test 4 has no additional weight added. The plots of Test 4.3 and Test 10 shown in Figures 6.7 and 6.17 are superimposed and shown below in Figure 7.11. The differences between the two are very prominent.


Figure 7.11 Superimposed plots of Test 4.3 (green) and Test 10 (blue).

First, the rise/fall curves are shifted 15-25 degrees to the right. This makes the active motor range about 45-100 degrees for +0.5 lbs . It is heavily skewed towards the top of the preferred motor range, and this is not ideal. This is compounded by the amount of meaningful motion past 90 degrees shown by Test 10. Ideally, $r_{l}$ would reach the top stop (maximum) before 90 degrees, and Test 10 fails that objective.

The curves of Test 10 have more change between loop iterations compared to Test 4. This is especially prominent on the falling curves. They are spaced out more, and the maximum height decreases with each loop. This is a strong indication of fatigue, which is unwanted.

Adding weight to the system shifts the active motor range past what is acceptable and increases the rate of fatigue. For these reasons, increased weight is not desirable for the system.

### 7.5 Twist Reversal Angle

The last parameter to analyze is the twist reversal angle. This is the point at which the motor switches directions. In every test discussed so far, the motor has switched directions at 120 steps and 1 step. To isolate this parameter, Tests 4,8 , and 9 will be compared. The plots for Test 4 are shown in Figures 6.5, 6.6, and 6.7. The plots for Tests 8 and 9 are shown in Figures 6.13 and 6.14 , respectively. Test 4 changes direction at 1 and 120 steps, Test 8 changes direction at 60 and 80 steps, and Test 9 changes direction at 35 and 75 steps. While Test 4 executes nine loops, Tests 8 and 9 execute only six.

The plot of Test 8 may look like one big, single loop, but it actually overlaps itself many times in the top right corner. A more descriptive visualization is provided below in Figure 7.12, where $r_{l}$ is plotted against time instead of motor position. The two plots together paint the complete picture. The platform starts at the minimum position, and then hits maximum by the time the motor reaches 80 steps. The motor then starts turning back and forth in a 20 step range, but the platform doesn't move until it leaves that range and starts heading back down to step 1 .


Figure 7.12 Alternate plot of Test 8. R1 is plotted over time (s).

Test 9 shows a more interesting plot. The platform starts to rise on each loop but doesn't make it to the top. When the motor reverses direction, the platform does not follow suit. Rather, it stays still for about 20 steps before finally starting to move back down. This is shown in Figure 6.14, but the overlap makes it hard to differentiate between loops. Below, in Figure 7.13, Test 9 is plotted against time. Here, the individual loops are clearly seen.


Figure 7.13 Alternate plot of Test 9. R1 is plotted over time (s).

The system is showing an interesting behavior. If the direction of the motor is reversed, the platform does not change position until some number of steps have been taken in the other direction. This happens regardless of where the platform is when the motor changes direction, or whether it was going up or down. This behavior indicates hysteresis in the system. At any moment, the system has a sensitive direction, and a resilient direction. This makes the system both hard to model and hard to control. However, improving this has proven to be difficult. This behavior is present in every test so far. Hysteresis results from energy being stored in the system somewhere. This stored energy could be from torsion in the PP beam, friction on the slider, or something else that is undetermined so far.

## CHAPTER 8: FUTURE WORK

The work presented in this thesis is only a part of the development of this mechanism. Going forward, there will be changes made and new phases of testing. Vibration isolation is the ultimate goal, and to achieve that there will need to be dynamic testing done. The mechanism was designed with this in mind, and the static tests have helped push it closer to the dynamic testing. In order to be successful, however, I believe some additional parameters need to be explored. First, the slider friction will be reduced either by adding lubrication or by switching out the slider. Second, the joint at the bearing will be kept from sticking either by redesign or adding a different lubrication. Hopefully, changing these two parameters will reduce the hysteresis of the system. Finally, the beam material will be switched from polypropylene to something with less creep, such as steel. This will extend the life and improve the reliability of the beam, and will hopefully reduce or eliminate the twisting problem. Polypropylene has fairly prominent hysteretic behavior (Zrida et al. 2009), and solely switching beam materials may minimize hysteresis to a manageable level.

For dynamic testing, I have considered two different vibration isolation approaches. The first will be a closed loop active control. Ideally this will emulate a "sky-hook", where the platform stays still relative to ground. The second approach involves tuning the mechanism such that the oscillatory motion is cancelled out by the deflection of the beam. The payload of the
mechanism behaves as a tuned mass damper and reduces harmonic vibrations. This assumes that the oscillations experienced by the mechanism are both sinusoidal and uniform.

Existing versions of vibration isolation technology employ semi-active suspension using the sky-hook concept (Savaresi \& Spelta 2007). This means that the payload maintains a stable position even when the mechanism itself is oscillating, as though it was supported by an imaginary hook in the sky. This is achieved through control of the damping and stiffness.

There has also been success in vibration isolation by using a tuned mass damper. This component stabilizes motion caused by harmonic vibration. It is usually calibrated for a specific frequency range, making it a passive control approach. However, through the use of variable stiffness and damping, Solase and others have made a mechanism that uses this concept and can adapt to different frequencies (Solase et. al. 2013).

## CHAPTER 9: THESIS CONTRIBUTIONS

Several things are now known about the system that were not known before. There seems to be a significant amount of hysteresis, which was not predicted. The cause is not certain, but it is certainly a property of the system. The polypropylene beams fatigue quickly, and another material may handle the cyclic loading better. The behavior of the system can be improved for controllability by raising the bottom stop, setting the top link length such that $R=1.2$, and reducing the weight of the top platform. This thesis has helped to understand the behavior of the system and pushed it towards its final goal, vibration isolation.

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## APPENDIX A: CAD DRAWINGS

This Appendix contains all technical drawings for the components of the mechanism and test apparatus.

# IECHNICAL DRAWINGS FOR MECHANISM AND TEST APPARATUS 



Figure A. 1 The front page of the technical drawings.

| PART LIST |  |  |  |
| :---: | :---: | :---: | :---: |
| PART ID | PART NAME | DESCRIPTION | QTY. |
| AA | Link | Actobotics 33-hole Aluminum Beam | 4 |
| $A B$ | Pillow Block | Actobotics Face Thru-Hole Pillow Block | 10 |
| AC | Compliant Beam | Laser-cut polypropylene beam | 1 |
| AD | Beam-Motor Coupling | 3D-printed coupling between beam and stepper motor | 1 |
| AE | Beam Top Fixture | 3D-printed fixture attached to top of beam and bottom of top links | 1 |
| AF | Slider | TSLOTS: Deluxe Double Flange Single Wide Linear Bearing - Short, \#657097 | 1 |
| AG | Channel | Actobotics Std. 3" Aluminum Channel | 3 |
| AH | Mount Plate | Cut from Actobotics 9'ㅈ12" Aluminum Pattern Plate, connects Mechanism to Test Apparatus | 2 |
| AJ | Washer | Std. \#6 flat washer | 38 |
| BA | Stepper Motor | Nema 34 CNC High Iorque Stepper Motor | 1 |
| BB | Extrusion (80cm) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 2 |
| BC | Extrusion (75cm) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 2 |
| BD | Extrusion (50cm) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 1 |
| BE | Extrusion ( 38 cm ) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 2 |
| BF | Extrusion (23cm) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 4 |
| BG | Extrusion (15cm) | TSLOTS: TS 15-15 GR, \#650080, Aluminum | 2 |
| BH | Corner Bracket | TSLOTS: 4 Hole Vertical Inside Corner Bracket, \#653135 | 7 |
| BJ | Corner Gusset | TSLOTS: 4 Hole Vertical Inside Corner Gusset, \#653169 | 16 |
| BK | Counterweight Plate | Actobotics 9' $\times 12^{\prime \prime}$ Aluminum Pattern Plate | 1 |
| BL | Mount Block | TSLOTS: 3 Hole Blank Base Plate, \#655660 | 1 |


A) MECHANISM

B TEST APPARATUS

Figure A. 2 The parts list for the technical drawings.


Figure A. 3 Views of just the mechanism with callouts labeling the different parts.

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Figure A. 4 Views of just the test apparatus with callouts labeling the different parts.


Figure A. 5 Drawings for the link, pillow block, and compliant beam.
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(AD) BEAM-MOTOR COUPLING


ALL DIMENSIONS IN INCHES

Figure A. 6 Drawing for the beam-motor coupling.
(AE) BEAM TOP FIXTURE


ALL DIMENSIONS IN INCHES
Figure A. 7 Drawing for the beam top fixture.
(AF) SLIDER


Figure A. 8 Drawing for the slider.


Figure A. 9 Drawings for the channel and mount plate.

（BF）EXTRUSION $(23 \mathrm{CM})$
（BE）EXTRUSION $(38 \mathrm{CM})$
管要
承要盆

（BD）EXTRUSION $(50 \mathrm{CM})$
第第

（BC）EXTRUSION $(75 \mathrm{CM})$
筧等


ALL DIMENSIONS IN INCHES
Figure A． 10 Drawings for the various lengths of extrusions．
(BH)
CORNER BRACKET

(B) CORNER GUSSET


Figure A. 11 Drawings for the corner bracket and corner gusset.
(BL) MOUNT BLOCK


Figure A. 12 Drawing for the mount block.

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## APPENDIX B: TESTING TABLES

This Appendix contains the tables similar to Table 7.1 for each test that had boxplots made of it. These tables are made from the test data and used to make the boxplots and midpoint plots shown in Chapter 7.

Table B. 1 Min, max, midpoint, and quartile values with their corresponding motor positions for the initial run of Test 1.

| Test 1.1 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.883 | 20.744 | 20.598 | 20.406 | 20.342 | 20.275 | 20.274 | 20.207 | 20.170 |
| Beam Angle | 111.1 | 115.1 | 114.6 | 113.9 | 115.2 | 115.1 | 116.3 | 114.1 | 114.4 |
| Min | 15.167 | 15.145 | 15.150 | 15.152 | 15.149 | 15.154 | 15.150 | 15.143 | 15.154 |
| Beam Angle | 87.1 | 88.3 | 89 | 88.2 | 90.9 | 91.1 | 91.8 | 91.6 | 92.1 |
| Midpoint | 18.025 | 17.945 | 17.874 | 17.779 | 17.746 | 17.715 | 17.712 | 17.675 | 17.662 |
| Beam Angle | 88.6 | 89.8 | 90.7 | 91.3 | 92.3 | 92.7 | 93.4 | 93.5 | 93.8 |
| 1/4 point | 16.596 | 16.545 | 16.512 | 16.466 | 16.447 | 16.435 | 16.431 | 16.409 | 16.408 |
| Beam Angle | 88.4 | 89.6 | 90.4 | 89.5 | 92.1 | 92.5 | 93.1 | 93.3 | 93.5 |
| 3/4 point | 19.454 | 19.345 | 19.236 | 19.093 | 19.044 | 18.995 | 18.993 | 18.941 | 18.916 |
| Beam Angle | 88.9 | 90.6 | 91.6 | 94.9 | 93.4 | 94.2 | 94.7 | 95.1 | 95.5 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.923 | 20.809 | 20.678 | 20.496 | 20.424 | 20.346 | 20.317 | 20.260 | 20.212 |
| Beam Angle | 59.3 | 72.5 | 76 | 76.9 | 76.9 | 79 | 78.3 | 79 | 81.4 |
| Min | 15.176 | 15.180 | 15.186 | 15.183 | 15.189 | 15.181 | 15.179 | 15.181 | 15.177 |
| Beam Angle | 37.2 | 40.6 | 43.5 | 46.3 | 47.6 | 48.6 | 49.7 | 50.6 | 51.5 |
| Midpoint | 18.050 | 17.995 | 17.932 | 17.839 | 17.807 | 17.764 | 17.748 | 17.721 | 17.694 |
| Beam Angle | 40.3 | 44.8 | 47.6 | 50.6 | 52.4 | 53.7 | 54.6 | 55.9 | 56.8 |
| 1/4 point | 16.613 | 16.588 | 16.559 | 16.511 | 16.498 | 16.472 | 16.463 | 16.451 | 16.436 |
| Beam Angle | 39.4 | 43.5 | 46.4 | 49.3 | 50.8 | 52.3 | 52.9 | 54.1 | 54.9 |
| 3/4 point | 19.486 | 19.402 | 19.305 | 19.167 | 19.116 | 19.055 | 19.032 | 18.990 | 18.953 |
| Beam Angle | 43.4 | 48.5 | 51.9 | 54.8 | 57 | 58.4 | 59.6 | 60.8 | 61.7 |

Table B. 2 Min, max, midpoint, and quartile values with their corresponding motor positions for the first replicate of Test 1.

Test 1.2

| Up |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.929 | 20.845 | 20.644 | 20.506 | 20.350 | 20.258 | 20.222 | 20.106 | 20.000 |
| Beam Angle | 111.3 | 117.4 | 118.2 | 120 | 120 | 120 | 120 | 120 | 120 |
| Min | 15.210 | 15.197 | 15.197 | 15.200 | 15.180 | 15.197 | 15.187 | 15.191 | 15.196 |
| Beam Angle | 88.9 | 89.8 | 92 | 92.3 | 91.6 | 92.2 | 93.3 | 93.2 | 92.1 |
| Midpoint | 18.070 | 18.021 | 17.920 | 17.853 | 17.765 | 17.727 | 17.704 | 17.648 | 17.598 |
| Beam Angle | 94.1 | 94.8 | 96.9 | 98.3 | 98.2 | 98.6 | 99.3 | 99.4 | 99.6 |
| 1/4 point | 16.640 | 16.609 | 16.558 | 16.526 | 16.473 | 16.462 | 16.446 | 16.419 | 16.397 |
| Beam Angle | 93.6 | 93.9 | 94.9 | 96.4 | 96.7 | 96.9 | 97.9 | 97.7 | 97.3 |
| 3/4 point | 19.499 | 19.433 | 19.282 | 19.179 | 19.057 | 18.993 | 18.963 | 18.877 | 18.799 |
| Beam Angle | 94.9 | 96 | 98.3 | 100 | 100.4 | 101 | 101.4 | 101.7 | 102.5 |


| Down |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Max | 20.953 | 20.889 | 20.727 | 20.603 | 20.466 | 20.355 | 20.328 | 20.222 |
| Beam Angle | 59.8 | 68.2 | 73.6 | 74.4 | 77 | 79.5 | 80.1 | 80.8 |
| Min | 15.232 | 15.229 | 15.212 | 15.216 | 15.227 | 15.205 | 15.209 | 15.216 |
| Beam Angle | 38.4 | 42.5 | 45.3 | 46.6 | 48.5 | 50.8 | 51 | 52.4 |
| Midpoint | 18.092 | 18.059 | 17.969 | 17.909 | 17.846 | 17.780 | 17.769 | 17.719 |
| Beam Angle | 39.9 | 44.6 | 48.2 | 50.5 | 52.5 | 54.9 | 55.3 | 56.9 |
| 1/4 point | 16.662 | 16.644 | 16.591 | 16.563 | 16.536 | 16.492 | 16.489 | 16.468 |
| Beam Angle | 39.1 | 43.6 | 46.8 | 48.8 | 50.8 | 52.7 | 53.2 | 54.7 |
| 3/4 point | 19.523 | 19.474 | 19.348 | 19.256 | 19.156 | 19.068 | 19.048 | 18.971 |
| Beam Angle | 42.9 | 48.7 | 52.9 | 55.7 | 58 | 59.8 | 60.1 | 62.9 |

Table B. 3 Min, max, midpoint, and quartile values with their corresponding motor positions for Test 2.

| Test 2 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.934 | 20.929 | 20.884 | 20.842 | 20.770 | 20.726 | 20.710 | 20.624 | 20.588 |
| Beam Angle | 97.2 | 108.9 | 116.3 | 116.4 | 114.1 | 116.6 | 120 | 120 | 120 |
| Min | 16.688 | 16.697 | 16.685 | 16.700 | 16.698 | 16.699 | 16.696 | 16.696 | 16.701 |
| Beam Angle | 76.2 | 76.2 | 77 | 76.7 | 77.2 | 77.7 | 78.2 | 77.8 | 78.3 |
| Midpoint | 18.811 | 18.813 | 18.784 | 18.771 | 18.734 | 18.713 | 18.703 | 18.660 | 18.644 |
| Beam Angle | 79.6 | 80.8 | 82.4 | 83.3 | 83.7 | 84.2 | 84.4 | 85.3 | 86.2 |
| 1/4 point | 17.750 | 17.755 | 17.734 | 17.735 | 17.716 | 17.706 | 17.699 | 17.678 | 17.672 |
| Beam Angle | 77.6 | 78 | 78.6 | 79.4 | 79.6 | 80.3 | 80.6 | 80.6 | 81.1 |
| 3/4 point | 19.873 | 19.871 | 19.834 | 19.806 | 19.752 | 19.720 | 19.706 | 19.642 | 19.616 |
| Beam Angle | 81.5 | 83.5 | 85.3 | 86.5 | 87.3 | 87.8 | 88.4 | 89 | 89.7 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.942 | 20.947 | 20.925 | 20.899 | 20.822 | 20.777 | 20.772 | 20.681 | 20.631 |
| Beam Angle | 55.4 | 61 | 68.1 | 71.5 | 75 | 74.1 | 76.2 | 78.2 | 76.9 |
| Min | 16.719 | 16.713 | 16.722 | 16.719 | 16.725 | 16.725 | 16.727 | 16.728 | 16.726 |
| Beam Angle | 38.3 | 40.5 | 42.2 | 43.2 | 44.1 | 45.1 | 45.8 | 46.4 | 46.9 |
| Midpoint | 18.831 | 18.830 | 18.824 | 18.809 | 18.773 | 18.751 | 18.750 | 18.705 | 18.678 |
| Beam Angle | 40.8 | 43.1 | 44.7 | 46.1 | 47.2 | 48 | 48.7 | 49.5 | 50.1 |
| 1/4 point | 17.775 | 17.772 | 17.773 | 17.764 | 17.749 | 17.738 | 17.739 | 17.717 | 17.702 |
| Beam Angle | 39.2 | 41.3 | 42.8 | 44.2 | 45.4 | 46.5 | 46.8 | 47.7 | 47.8 |
| 3/4 point | 19.887 | 19.888 | 19.874 | 19.854 | 19.797 | 19.764 | 19.761 | 19.693 | 19.654 |
| Beam Angle | 42.9 | 46 | 48.8 | 50.5 | 52.4 | 53.4 | 54.1 | 55.5 | 56.6 |

Table B. 4 Min, max, midpoint, and quartile values with their corresponding motor positions for Test 3.

| Test 3 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.184 | 20.188 | 20.164 | 20.130 | 20.082 | 20.035 | 20.083 | 19.998 | 19.946 |
| Beam Angle | 95.7 | 99.2 | 105.8 | 108.4 | 111.2 | 119.7 | 120 | 120 | 120 |
| Min | 16.701 | 16.699 | 16.700 | 16.696 | 16.698 | 16.704 | 16.696 | 16.699 | 16.703 |
| Beam Angle | 76.1 | 76.2 | 76.2 | 77.4 | 78.3 | 79.1 | 79.3 | 79.1 | 79.2 |
| Midpoint | 18.442 | 18.443 | 18.432 | 18.413 | 18.390 | 18.369 | 18.389 | 18.349 | 18.324 |
| Beam Angle | 78.2 | 78.3 | 79.2 | 80.3 | 81.8 | 82.5 | 83 | 83.2 | 83.2 |
| 1/4 point | 17.572 | 17.571 | 17.566 | 17.554 | 17.544 | 17.536 | 17.543 | 17.524 | 17.513 |
| Beam Angle | 77.7 | 77.6 | 78.3 | 79.3 | 80.9 | 81.6 | 82.3 | 82.2 | 82.1 |
| 3/4 point | 19.313 | 19.316 | 19.298 | 19.272 | 19.236 | 19.202 | 19.236 | 19.173 | 19.135 |
| Beam Angle | 80.3 | 81 | 82.6 | 84.1 | 85.5 | 86.5 | 86.6 | 87.4 | 87.8 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.195 | 20.199 | 20.191 | 20.176 | 20.141 | 20.097 | 20.137 | 20.057 | 20.005 |
| Beam Angle | 53.2 | 59.2 | 64.3 | 67 | 72.8 | 74.5 | 74.8 | 74.1 | 72.4 |
| Min | 16.727 | 16.731 | 16.725 | 16.725 | 16.727 | 16.727 | 16.725 | 16.729 | 16.731 |
| Beam Angle | 33.6 | 36.2 | 38 | 37.2 | 38.3 | 39.6 | 39.4 | 41 | 41.6 |
| Midpoint | 18.461 | 18.465 | 18.458 | 18.451 | 18.434 | 18.412 | 18.431 | 18.393 | 18.368 |
| Beam Angle | 38.5 | 41.6 | 43.3 | 43.8 | 45.6 | 46.7 | 46.5 | 47.6 | 48.7 |
| 1/4 point | 17.594 | 17.598 | 17.591 | 17.588 | 17.581 | 17.569 | 17.578 | 17.561 | 17.550 |
| Beam Angle | 35.3 | 38.1 | 39.7 | 39.6 | 40.8 | 42 | 42 | 43 | 44.2 |
| 3/4 point | 19.328 | 19.332 | 19.324 | 19.314 | 19.288 | 19.254 | 19.284 | 19.225 | 19.187 |
| Beam Angle | 42 | 45.4 | 47.5 | 48.8 | 50.5 | 51.8 | 51.6 | 53 | 54 |

Table B. 5 Min, max, midpoint, and quartile values with their corresponding motor positions for the first replicate of Test 4.

| Test 4.2 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.944 | 20.937 | 20.939 | 20.936 | 20.942 | 20.936 | 20.941 | 20.934 | 20.938 |
| Beam Angle | 76.7 | 83 | 84.4 | 87.7 | 89.2 | 91.2 | 92.2 | 95.2 | 95.9 |
| Min | 18.223 | 18.225 | 18.231 | 18.226 | 18.219 | 18.231 | 18.225 | 18.216 | 18.228 |
| Beam Angle | 63.9 | 65.3 | 65.3 | 66.2 | 66 | 66.3 | 67.2 | 66.4 | 66.2 |
| Midpoint | 19.584 | 19.581 | 19.585 | 19.581 | 19.580 | 19.584 | 19.583 | 19.575 | 19.583 |
| Beam Angle | 66.2 | 67.5 | 68.2 | 69.3 | 69.3 | 69.4 | 70.2 | 69.9 | 69.9 |
| 1/4 point | 18.903 | 18.903 | 18.908 | 18.904 | 18.900 | 18.907 | 18.904 | 18.895 | 18.905 |
| Beam Angle | 65.6 | 66.9 | 67.3 | 68.3 | 68.3 | 68.3 | 69.1 | 68.7 | 68.6 |
| 3/4 point | 20.264 | 20.259 | 20.262 | 20.259 | 20.261 | 20.260 | 20.262 | 20.254 | 20.260 |
| Beam Angle | 67.9 | 69.8 | 71.1 | 71.7 | 72.3 | 72.7 | 73.4 | 74 | 74 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.965 | 20.964 | 20.961 | 20.957 | 20.956 | 20.949 | 20.952 | 20.953 | 20.942 |
| Beam Angle | 43.9 | 49.3 | 50.9 | 52.9 | 54.9 | 58.8 | 58.8 | 58.5 | 59.7 |
| Min | 18.246 | 18.245 | 18.234 | 18.247 | 18.252 | 18.237 | 18.245 | 18.249 | 18.239 |
| Beam Angle | 31.4 | 33.1 | 34.1 | 35.3 | 35.8 | 35.9 | 36.8 | 37.3 | 37.7 |
| Midpoint | 19.606 | 19.605 | 19.598 | 19.602 | 19.604 | 19.593 | 19.599 | 19.601 | 19.591 |
| Beam Angle | 32.6 | 34.6 | 36.1 | 37.1 | 38 | 38.5 | 38.6 | 39.4 | 39.7 |
| 1/4 point | 18.926 | 18.925 | 18.916 | 18.925 | 18.928 | 18.915 | 18.922 | 18.925 | 18.915 |
| Beam Angle | 32.1 | 34 | 35.5 | 36.2 | 36.7 | 37.5 | 37.7 | 38.3 | 38.6 |
| 3/4 point | 20.285 | 20.284 | 20.280 | 20.280 | 20.280 | 20.271 | 20.275 | 20.277 | 20.267 |
| Beam Angle | 34.7 | 37.2 | 38.8 | 40 | 40.9 | 41.8 | 42.5 | 43.1 | 43.6 |

Table B. 6 Min, max, midpoint, and quartile values with their corresponding motor positions for the second replicate of Test 4.

| Test 4.3 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.942 | 20.934 | 20.939 | 20.936 | 20.936 | 20.930 | 20.931 | 20.934 | 20.935 |
| Beam Angle | 79.8 | 86.4 | 90.3 | 94.8 | 99 | 100.3 | 101.7 | 104.8 | 109.3 |
| Min | 18.222 | 18.213 | 18.218 | 18.212 | 18.204 | 18.218 | 18.211 | 18.206 | 18.221 |
| Beam Angle | 63 | 64.7 | 64.6 | 66.2 | 67.1 | 67.8 | 67.2 | 67.5 | 67.4 |
| Midpoint | 19.582 | 19.573 | 19.578 | 19.574 | 19.570 | 19.574 | 19.571 | 19.570 | 19.578 |
| Beam Angle | 67.5 | 69 | 69.8 | 71.2 | 71.7 | 72.1 | 72.4 | 72.5 | 73.2 |
| 1/4 point | 18.902 | 18.893 | 18.898 | 18.893 | 18.887 | 18.896 | 18.891 | 18.888 | 18.899 |
| Beam Angle | 66.7 | 67.4 | 68.3 | 69.6 | 70.3 | 70.5 | 70.8 | 70.9 | 71.3 |
| 3/4 point | 20.262 | 20.253 | 20.259 | 20.255 | 20.253 | 20.252 | 20.251 | 20.252 | 20.256 |
| Beam Angle | 69.6 | 72 | 73.2 | 74.8 | 76.2 | 76.4 | 77.1 | 77.3 | 77.3 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.949 | 20.945 | 20.944 | 20.944 | 20.942 | 20.940 | 20.943 | 20.945 | 20.946 |
| Beam Angle | 46.1 | 50.9 | 54.1 | 55 | 59.8 | 60.9 | 60.5 | 64 | 65.1 |
| Min | 18.236 | 18.237 | 18.227 | 18.224 | 18.236 | 18.227 | 18.233 | 18.237 | 18.237 |
| Beam Angle | 31.9 | 34.3 | 35.4 | 36.3 | 37.3 | 37.5 | 37.7 | 38 | 38.5 |
| Midpoint | 19.592 | 19.591 | 19.586 | 19.584 | 19.589 | 19.584 | 19.588 | 19.591 | 19.591 |
| Beam Angle | 33.4 | 35.7 | 37.6 | 38.6 | 39.5 | 40.1 | 40.7 | 41.3 | 41.7 |
| 1/4 point | 18.914 | 18.914 | 18.906 | 18.904 | 18.913 | 18.905 | 18.910 | 18.914 | 18.914 |
| Beam Angle | 32.6 | 35.2 | 36.5 | 37.7 | 38.5 | 39 | 39.5 | 39.8 | 40.2 |
| 3/4 point | 20.271 | 20.268 | 20.265 | 20.264 | 20.265 | 20.262 | 20.265 | 20.268 | 20.268 |
| Beam Angle | 34.8 | 38.5 | 40.5 | 41.3 | 42.5 | 43 | 43.7 | 44.4 | 44.8 |

Table B. 7 Min, max, midpoint, and quartile values with their corresponding motor positions for the initial run of Test 7.

| Test 7.1 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.939 | 20.927 | 20.937 | 20.931 | 20.929 | 20.942 | 20.932 | 20.939 | 20.931 |
| Beam Angle | 74.8 | 75.7 | 80.1 | 81 | 82.7 | 84.5 | 84.1 | 85.2 | 86.1 |
| Min | 18.205 | 18.197 | 18.202 | 18.192 | 18.195 | 18.195 | 18.193 | 18.191 | 18.192 |
| Beam Angle | 66.3 | 67.8 | 68.1 | 69.4 | 68.7 | 68.7 | 69.2 | 68.8 | 69.4 |
| Midpoint | 19.572 | 19.562 | 19.569 | 19.561 | 19.562 | 19.568 | 19.563 | 19.565 | 19.562 |
| Beam Angle | 71.8 | 72.8 | 72.5 | 74 | 73.5 | 73.9 | 74.6 | 74.1 | 74.3 |
| 1/4 point | 18.888 | 18.879 | 18.886 | 18.877 | 18.879 | 18.882 | 18.878 | 18.878 | 18.877 |
| Beam Angle | 70.7 | 71.7 | 71.4 | 73 | 72.3 | 72.7 | 73.5 | 72.9 | 73.2 |
| 3/4 point | 20.256 | 20.244 | 20.253 | 20.246 | 20.246 | 20.255 | 20.248 | 20.252 | 20.246 |
| Beam Angle | 72.6 | 73.5 | 73.3 | 74.7 | 74.4 | 74.7 | 75.5 | 75 | 75.2 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.953 | 20.940 | 20.945 | 20.940 | 20.941 | 20.942 | 20.938 | 20.940 | 20.948 |
| Beam Angle | 43.6 | 46.5 | 48.3 | 49.3 | 50.7 | 50.9 | 51.9 | 53.1 | 53.3 |
| Min | 18.222 | 18.220 | 18.339 | 18.218 | 18.220 | 18.332 | 18.219 | 18.222 | 18.340 |
| Beam Angle | 27.7 | 28.9 | 29.5 | 30.8 | 31.3 | 31.8 | 32.2 | 32 | 32.3 |
| Midpoint | 19.588 | 19.580 | 19.642 | 19.579 | 19.580 | 19.637 | 19.578 | 19.581 | 19.644 |
| Beam Angle | 31.3 | 32.7 | 33.7 | 34.5 | 35.1 | 36 | 35.9 | 36.6 | 37.2 |
| 1/4 point | 18.905 | 18.900 | 18.991 | 18.899 | 18.900 | 18.985 | 18.899 | 18.902 | 18.992 |
| Beam Angle | 29.8 | 31.1 | 32.1 | 32.9 | 33.5 | 33.9 | 34.2 | 34.6 | 35 |
| 3/4 point | 20.271 | 20.260 | 20.293 | 20.260 | 20.261 | 20.290 | 20.258 | 20.261 | 20.296 |
| Beam Angle | 34.3 | 35.7 | 36.7 | 37.9 | 38.7 | 39.5 | 39.6 | 40.4 | 40.6 |

Table B. 8 Min, max, midpoint, and quartile values with their corresponding motor positions for the first replicate of Test 7.

| Test 7.2 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.933 | 20.938 | 20.936 | 20.927 | 20.934 | 20.936 | 20.934 | 20.938 | 20.938 |
| Beam Angle | 79 | 79.7 | 81.4 | 83.5 | 84.9 | 86.8 | 86 | 87.8 | 88.6 |
| Min | 18.208 | 18.211 | 18.207 | 18.206 | 18.207 | 18.207 | 18.210 | 18.209 | 18.206 |
| Beam Angle | 62.8 | 65.1 | 65.1 | 65.8 | 65.8 | 66.8 | 67.1 | 67.3 | 66.8 |
| Midpoint | 19.571 | 19.574 | 19.571 | 19.566 | 19.570 | 19.572 | 19.572 | 19.573 | 19.572 |
| Beam Angle | 67.9 | 69.5 | 70 | 71.7 | 71.5 | 71.8 | 72.6 | 72.2 | 72.5 |
| 1/4 point | 18.889 | 18.893 | 18.889 | 18.886 | 18.889 | 18.889 | 18.891 | 18.891 | 18.889 |
| Beam Angle | 66.7 | 68.2 | 68.6 | 70.4 | 70.2 | 70.5 | 71.3 | 70.9 | 71.1 |
| 3/4 point | 20.252 | 20.256 | 20.253 | 20.247 | 20.252 | 20.254 | 20.253 | 20.256 | 20.255 |
| Beam Angle | 68.9 | 70.6 | 71.1 | 72.6 | 72.5 | 72.9 | 73.6 | 73.4 | 73.6 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.943 | 20.943 | 20.944 | 20.943 | 20.940 | 20.940 | 20.944 | 20.941 | 20.940 |
| Beam Angle | 42.6 | 45.6 | 48.5 | 51 | 50.1 | 52 | 51.8 | 53.8 | 54.3 |
| Min | 18.230 | 18.228 | 18.345 | 18.224 | 18.225 | 18.345 | 18.226 | 18.227 | 18.343 |
| Beam Angle | 26.5 | 27.8 | 29.3 | 30.7 | 31.4 | 30.9 | 31.5 | 31.5 | 31.3 |
| Midpoint | 19.586 | 19.586 | 19.645 | 19.584 | 19.582 | 19.643 | 19.585 | 19.584 | 19.642 |
| Beam Angle | 30.9 | 32.2 | 33.2 | 34.5 | 35.1 | 35.6 | 35.8 | 36.1 | 36.7 |
| 1/4 point | 18.908 | 18.907 | 18.995 | 18.904 | 18.904 | 18.994 | 18.905 | 18.905 | 18.992 |
| Beam Angle | 29.5 | 31 | 31.9 | 33 | 33.5 | 33.9 | 34 | 34.4 | 34.7 |
| 3/4 point | 20.264 | 20.264 | 20.294 | 20.264 | 20.261 | 20.292 | 20.264 | 20.263 | 20.291 |
| Beam Angle | 33.7 | 35.3 | 36.3 | 37.6 | 38.1 | 38.9 | 39.2 | 39.7 | 40.2 |

Table B. 9 Min, max, midpoint, and quartile values with their corresponding motor positions for the second replicate of Test 7.

| Test 7.3 |  |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  | Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| Max | 20.945 | 20.939 | 20.932 | 20.935 | 20.933 | 20.930 | 20.935 | 20.932 | 20.937 |  |
| Beam Angle | 79.5 | 84.4 | 87.2 | 90 | 92.5 | 94.7 | 93.6 | 96.7 | 97.6 |  |
| Min | 18.220 | 18.211 | 18.215 | 18.205 | 18.208 | 18.208 | 18.206 | 18.209 | 18.210 |  |
| Beam Angle | 62 | 64 | 65 | 66.6 | 65.7 | 67.4 | 68.6 | 68.3 | 67.7 |  |
| Midpoint | 19.583 | 19.575 | 19.573 | 19.570 | 19.570 | 19.569 | 19.570 | 19.570 | 19.573 |  |
| Beam Angle | 68.7 | 70.1 | 71 | 72.8 | 72.9 | 73.5 | 74.3 | 74.1 | 74.2 |  |
| 1/4 point | 18.901 | 18.893 | 18.894 | 18.887 | 18.889 | 18.888 | 18.888 | 18.890 | 18.891 |  |
| Beam Angle | 67.4 | 68.2 | 69.2 | 71.1 | 71 | 72.1 | 72.9 | 72.5 | 72.5 |  |
| 3/4 point | 20.264 | 20.257 | 20.253 | 20.252 | 20.252 | 20.250 | 20.253 | 20.251 | 20.255 |  |
| Beam Angle | 70.1 | 71.6 | 72.6 | 74.1 | 74.3 | 75.1 | 75.8 | 76 | 76.3 |  |
|  |  |  |  | Down |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| Max | 20.965 | 20.945 | 20.948 | 20.942 | 20.944 | 20.940 | 20.940 | 20.941 | 20.942 |  |
| Beam Angle | 47 | 50.4 | 52.7 | 54.1 | 56.6 | 58.3 | 58.4 | 59.4 | 61.8 |  |
| Min | 18.231 | 18.230 | 18.348 | 18.223 | 18.232 | 18.346 | 18.227 | 18.228 | 18.343 |  |
| Beam Angle | 29.5 | 30.4 | 31.2 | 31.9 | 32.7 | 32.7 | 33.5 | 34.2 | 34.4 |  |
| Midpoint | 19.598 | 19.587 | 19.648 | 19.583 | 19.588 | 19.643 | 19.583 | 19.584 | 19.643 |  |
| Beam Angle | 32.9 | 34.7 | 36 | 37 | 37.8 | 38.3 | 38.4 | 39.1 | 39.7 |  |
| 1/4 point | 18.914 | 18.909 | 18.998 | 18.903 | 18.910 | 18.994 | 18.905 | 18.906 | 18.993 |  |
| Beam Angle | 31.3 | 33 | 34.1 | 35.1 | 35.5 | 35.9 | 36.1 | 36.5 | 36.9 |  |
| 3/4 point | 20.282 | 20.266 | 20.298 | 20.263 | 20.266 | 20.291 | 20.262 | 20.263 | 20.292 |  |
| Beam Angle | 35.9 | 38 | 39.6 | 41.1 | 41.8 | 42.4 | 42.4 | 43.2 | 43.8 |  |

Table B. 10 Min, max, midpoint, and quartile values with their corresponding motor positions for Test 10.

| Test 10 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Up |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.805 | 20.436 | 20.177 | 20.046 | 19.813 | 19.685 | 19.688 | 19.537 | 19.486 |
| Beam Angle | 112 | 111.9 | 113.7 | 110.2 | 109.6 | 107.6 | 109.8 | 110 | 110.8 |
| Min | 18.218 | 18.212 | 18.218 | 18.211 | 18.211 | 18.215 | 18.212 | 18.210 | 18.214 |
| Beam Angle | 80.2 | 82.3 | 83.1 | 84.7 | 85.5 | 85.1 | 85.6 | 86.4 | 86.5 |
| Midpoint | 19.511 | 19.324 | 19.197 | 19.129 | 19.012 | 18.950 | 18.950 | 18.874 | 18.850 |
| Beam Angle | 85.7 | 88.5 | 89.6 | 90.8 | 91.4 | 92 | 92.4 | 92.7 | 93.4 |
| 1/4 point | 18.864 | 18.768 | 18.707 | 18.670 | 18.611 | 18.583 | 18.581 | 18.542 | 18.532 |
| Beam Angle | 82.8 | 85.9 | 87.2 | 88.4 | 89.2 | 89.1 | 89.2 | 89.5 | 90 |
| 3/4 point | 20.158 | 19.880 | 19.687 | 19.587 | 19.412 | 19.318 | 19.319 | 19.206 | 19.168 |
| Beam Angle | 91.9 | 94.4 | 96.3 | 97.6 | 96.8 | 96.6 | 97.7 | 97.1 | 97.8 |
| Down |  |  |  |  |  |  |  |  |  |
| Iteration | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Max | 20.870 | 20.571 | 20.275 | 20.147 | 19.913 | 19.748 | 19.727 | 19.601 | 19.518 |
| Beam Angle | 70.6 | 77.6 | 78.5 | 80.7 | 83.8 | 85.4 | 85.1 | 84.7 | 84.9 |
| Min | 18.229 | 18.230 | 18.227 | 18.226 | 18.228 | 18.228 | 18.229 | 18.227 | 18.225 |
| Beam Angle | 51.2 | 55.7 | 58.5 | 59.7 | 61.8 | 63.4 | 63.9 | 65.5 | 66.2 |
| Midpoint | 19.549 | 19.400 | 19.251 | 19.187 | 19.070 | 18.988 | 18.978 | 18.914 | 18.871 |
| Beam Angle | 53.5 | 59.5 | 62.6 | 64.1 | 66.5 | 68.7 | 69.7 | 70.3 | 71.2 |
| 1/4 point | 18.889 | 18.815 | 18.739 | 18.706 | 18.649 | 18.608 | 18.604 | 18.570 | 18.548 |
| Beam Angle | 52.1 | 56.8 | 59.7 | 61 | 63.6 | 65.6 | 66.2 | 67.4 | 68 |
| 3/4 point | 20.210 | 19.986 | 19.763 | 19.667 | 19.492 | 19.368 | 19.352 | 19.257 | 19.194 |
| Beam Angle | 56.6 | 63 | 66.8 | 68.7 | 71.3 | 73.8 | 74.3 | 75.6 | 76.6 |

## APPENDIX C: ARDUINO CODE

This appendix contains all the Arduino code used for testing.

## C. 1 Tests 1-4, 10

```
int ena = 11;
int dir = 12;
int puls = 13;
int sensA = A0;
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int e1 = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;
int avg = 0;
void setup()
{
    pinMode(ena, OUTPUT);
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
}
void loop() {
    int steps = 1;
    for(int z = 0; z < 3; z++)
{
    digitalWrite(dir,HIGH); //ccw
    int direc = 1;
    for(int y = 0; y < 119; y++)
```

```
{
    int iter = 0;
    for(int x = 0; x < 10; x++)
    {
        for (int j=0; j<10; j++)
        {
            e5=e4;
            e4=e3;
            e3=e2;
            e2=e1;
            e1=e;
            e=d;
            d=c;
            c=b;
            b=a;
            a = analogRead(sensB);
        }
        avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
        //channel[0] = analogRead(sensA);
        channel[1] = analogRead(sensB);
        Serial.print(moPos);
        Serial.print("\t");
        Serial.print(moPos+(iter*.1*direc));
        Serial.print("\t");
        Serial.println(avg);
        iter = iter + 1;
        delay(100);
    }
    for(int x = 0; x < steps; x++)
    {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
    }
    moPos = moPos + steps;
}
digitalWrite(dir, LOW); //cw
direc = -1;
for(int y = 0; y < 119; y++)
{
    int iter = 0;
    for(int x = 0; x < 10; x++)
    {
        for (int j=0; j<10; j++)
        {
            e5=e4;
            e4=e3;
            e3=e2;
            e2=e1;
            e1=e;
            e=d;
            d=c;
            c=b ;
```

```
                b=a;
                a = analogRead(sensB);
            }
            avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(100);
        }
        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
}
    exit(0);
}
```


## C. 2 Test 5

```
int ena = 11;
int dir = 12;
int puls = 13;
int sensA = A0;
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int e1 = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;
int avg = 0;
```

void setup()
\{
pinMode(ena, OUTPUT);

```
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
}
void loop() {
    int steps = 1;
        for(int z = 0; z < 3; z++)
        {
            digitalWrite(dir,HIGH); //ccw
            int direc = 1;
            for(int }\textrm{y}=0;\textrm{Y}<119; Y++
        {
            int iter = 0;
            for(int }\textrm{x}=0;\textrm{x}< < 10; x++
            {
                for (int j=0; j<10; j++)
                {
                    e5=e4;
                    e4=e3;
                        e3=e2;
                    e2=e1;
                    e1=e;
                    e=d;
                    d=c;
                    c=b;
                    b=a;
                    a = analogRead(sensB);
                }
            avg=(a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc)) ;
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(50);
            }
            for(int x = 0; x < steps; x++)
            {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
            }
            moPos = moPos + steps;
        }
```

```
    digitalWrite(dir, LOW); //cw
    direc = -1;
    for(int y = 0; y < 119; y++)
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
            {
                    e5=e4;
                    e4=e3;
                    e3=e2;
                    e2=e1;
                    e1=e;
                    e=d;
                    d=c;
                    c=b;
                    b=a;
                    a = analogRead(sensB);
            }
            avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(50);
        }
        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
    }
    exit(0);
}
```


## C. 3 Test 6

```
int ena = 11;
int dir = 12;
int puls = 13;
```

int sensA $=A 0$;

```
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int e1 = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;
int avg = 0;
void setup()
{
    pinMode(ena, OUTPUT);
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
}
void loop() {
    int steps = 1;
        for(int z = 0; z < 3; z++)
        {
            digitalWrite(dir,HIGH); //ccw
            int direc = 1;
            for(int y = 0; y < 119; y++)
        {
            int iter = 0;
            for(int x = 0; x < 10; x++)
            {
                for (int j=0; j<10; j++)
                    {
                        e5=e4;
                        e4=e3;
                        e3=e2;
                        e2=e1;
                        e1=e;
                        e=d;
                        d=c;
                    c=b;
                        b=a;
                        a = analogRead(sensB);
            }
                avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
                //channel[0] = analogRead(sensA);
```

            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(25);
    }
    for(int x = 0; x < steps; x++)
    {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
    }
    moPos = moPos + steps;
    }
digitalWrite(dir, LOW); //cw
direc = -1;
for(int y = 0; y < 119; y++)
{
int iter = 0;
for(int x = 0; x < 10; x++)
{
for (int j=0; j<10; j++)
{
e5=e4;
e4=e3;
e3=e2;
e2=e1;
e1=e;
e=d;
d=c;
c=b;
b=a;
a = analogRead(sensB);
}
avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
//channel[0] = analogRead(sensA);
channel[1] = analogRead(sensB);
Serial.print(moPos);
Serial.print("\t");
Serial.print(moPos+(iter*.1*direc));
Serial.print("\t");
Serial.println(avg);
iter = iter + 1;
delay(25);
}
for(int x = 0; x < steps; x++)
{
digitalWrite(puls, HIGH);
delay(1);

```
```

                digitalWrite(puls, LOW); // emits a 2ms pulse, advancing l step
                delay(1);
        }
        moPos = moPos - steps;
        }
    }
    exit(0);
    }

```

\section*{C. 4 Test 7}
```

int ena = 11;
int dir = 12;
int puls = 13;
int sensA = A0;
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int e1 = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;
int avg = 0;

```
void setup()
\{
    pinMode(ena, OUTPUT);
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
\}
void loop() \{
    int steps = 1;
        for (int \(z=0 ; z<3 ; \quad z++)\)
        \{
            digitalWrite(dir,HIGH); //ccw
            int direc = 1;
        for (int \(y=0 ; y<119 ; ~ y++)\)
```

    int iter = 0;
    for(int x = 0; x < 10; x++)
    {
        for (int j=0; j<10; j++)
        {
            e5=e4;
            e4=e3;
            e3=e2;
            e2=e1;
            e1=e;
            e=d;
            d=c;
            c=b;
            b=a;
            a = analogRead(sensB);
        }
        avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
        //channel[0] = analogRead(sensA);
        channel[1] = analogRead(sensB);
        Serial.print(moPos);
        Serial.print("\t");
        Serial.print(moPos+(iter*.1*direc));
        Serial.print("\t");
        Serial.println(avg);
        iter = iter + 1;
        delay(10);
    }
    for(int x = 0; x < steps; x++)
    {
            digitalWrite(puls, HIGH);
            delay(1);
        digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
        delay(1);
    }
    moPos = moPos + steps;
    }
digitalWrite(dir, LOW); //cw
direc = -1;
for(int y = 0; y < 119; y++)
{
int iter = 0;
for(int x = 0; x < 10; x++)
{
for (int j=0; j<10; j++)
{
e5=e4;
e4=e3;
e3=e2;
e2=e1;
e1=e;
e=d;
d=c;
c=b;
b=a;

```
```

                a = analogRead(sensB);
                }
                avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
                //channel[0] = analogRead(sensA);
                channel[1] = analogRead(sensB);
                Serial.print(moPos);
                Serial.print("\t");
                Serial.print(moPos+(iter*.1*direc));
                Serial.print("\t");
                Serial.println(avg);
                iter = iter + 1;
                delay(10);
        }
        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
    }
    exit(0);
    }

```

\section*{C. 5 Test 8}
```

int ena = 11;
int dir = 12;
int puls = 13;
int sensA = A0;
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int e1 = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;

```
int avg \(=0\);
void setup()
\{
    pinMode(ena, OUTPUT);
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
```

    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
    }
void loop() {
int steps = 1;
//up to hys loop beginning
digitalWrite(dir,HIGH); //ccw
int direc = 1;
for(int y = 0; y < 79; y++)
{
int iter = 0;
for(int x = 0; x < 10; x++)
{
for (int j=0; j<10; j++)
{
e5=e4;
e4=e3;
e3=e2;
e2=e1;
e1=e;
e=d;
d=c;
c=b;
b=a;
a = analogRead(sensB);
}
avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
//channel[0] = analogRead(sensA);
channel[1] = analogRead(sensB);
Serial.print(moPos);
Serial.print("\t");
Serial.print(moPos+(iter*.1*direc));
Serial.print("\t");
Serial.println(avg);
iter = iter + 1;
delay(100);
}
for(int x = 0; x < steps; x++)
{
digitalWrite(puls, HIGH);
delay(1);
digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
delay(1);
}
moPos = moPos + steps;
}
//start hys loop
for(int z = 0; z < 6; z++)
{

```
        digitalWrite(dir, LOW); //cw
        direc = -1;
    for(int y = 0; y < 20; y++)
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
            {
                e5=e4;
                    e4=e3;
                    e3=e2;
                    e2=e1;
                    e1=e;
                    e=d;
                    d=c;
                    c=b;
                        b=a;
                a = analogRead(sensB);
            }
            avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(100);
        }
            for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
//up
    digitalWrite(dir,HIGH); //ccw
    int direc = 1;
    for(int y = 0; y < 20; y++)
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
            {
            e5=e4;
            e4=e3;
            e3=e2;
```

```
                e2=e1;
                e1=e;
                    e=d;
                    d=c;
                    c=b;
                    b=a;
                    a = analogRead(sensB);
                }
                avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
                //channel[0] = analogRead(sensA);
                channel[1] = analogRead(sensB);
                Serial.print(moPos);
                Serial.print("\t");
                Serial.print(moPos+(iter*.1*direc));
                Serial.print("\t");
                Serial.println(avg);
                iter = iter + 1;
                delay(100);
                }
            for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing l step
            delay(1);
        }
        moPos = moPos + steps;
        }
    }
//end loop, back down
    digitalWrite(dir, LOW); //cw
    direc = -1;
    for(int y = 0; y < 79; y++)
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
            {
            e5=e4;
            e4=e3;
            e3=e2;
            e2=e1;
            e1=e;
            e=d;
            d=c;
            c=b;
            b=a;
            a = analogRead(sensB);
        }
        avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
        //channel[0] = analogRead(sensA);
        channel[1] = analogRead(sensB);
        Serial.print(moPos);
```

```
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(100);
        }
        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
    exit(0);
}
```


## C. 6 Test 9

```
int ena = 11;
int dir = 12;
int puls = 13;
int sensA = A0;
int sensB = A1;
int channel[2];
int moPos = 1;
int a = 0;
int b = 0;
int c = 0;
int d = 0;
int e = 0;
int el = 0;
int e2 = 0;
int e3 = 0;
int e4 = 0;
int e5 = 0;
int avg = 0;
void setup()
{
    pinMode(ena, OUTPUT);
    pinMode(dir, OUTPUT);
    pinMode(puls, OUTPUT);
    pinMode(sensA, INPUT);
    pinMode(sensB, INPUT);
    digitalWrite(ena, LOW); //activates motor
    digitalWrite(dir,HIGH); //ccw
    Serial.begin(9600);
```

```
void loop() {
    int steps = 1;
//up to hys loop beginning
    digitalWrite(dir,HIGH); //ccw
    int direc = 1;
    for(int y = 0; y < 74; y++)
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
                {
                    e5=e4;
                    e4=e3;
                    e3=e2;
                    e2=e1;
                    e1=e;
                    e=d;
                    d=c;
                    c=b;
                    b=a;
                    a = analogRead(sensB);
            }
            avg = (a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print(moPos+(iter*.1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(100);
        }
            for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos + steps;
    }
//start hysteresis loop
    for(int z = 0; z < 6; z++)
    {
        digitalWrite(dir, LOW); //cw
        direc = -1;
        for(int y = 0; y < 40; y++)
        {
            int iter = 0;
            for(int x = 0; x < 10; x++)
```

```
        {
            for (int j=0; j<10; j++)
                {
                    e5=e4;
                    e4=e3;
                        e3=e2;
                        e2=e1;
                        e1=e;
                        e=d;
                    d=c;
                    c=b;
                    b=a;
                    a = analogRead(sensB);
            }
            avg=(a+b+c+d+e+e1+e2+e3+e4+e5)/10;
            //channel[0] = analogRead(sensA);
            channel[1] = analogRead(sensB);
            Serial.print(moPos);
            Serial.print("\t");
            Serial.print (moPos+(iter*. 1*direc));
            Serial.print("\t");
            Serial.println(avg);
            iter = iter + 1;
            delay(100);
        }
            for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
//up
    digitalWrite(dir,HIGH); //ccw
    int direc = 1;
    for(int }y=0;y<40; y++
    {
        int iter = 0;
        for(int x = 0; x < 10; x++)
        {
            for (int j=0; j<10; j++)
            {
            e5=e4;
            e4=e3;
            e3=e2;
            e2=e1;
            e1=e;
            e=d;
            d=c;
            c=b;
            b=a;
            a = analogRead(sensB);
```

                }
                avg=(a+b+c+d+e+e1+e2+e3+e4+e5)/10;
                //channel[0] = analogRead(sensA);
                channel[1] = analogRead(sensB);
                Serial.print(moPos);
                Serial.print("\t");
                Serial.print(moPos+(iter*.1*direc));
                Serial.print("\t");
                Serial.println(avg);
                iter = iter + 1;
                delay(100);
        }
        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos + steps;
        }
    }
    //end of loop, back down
digitalWrite(dir, LOW); //cw
direc = -1;
for(int }Y=0;y<74; y++
{
int iter = 0;
for(int }x=0;x<10; x++
{
for (int j=0; j<10; j++)
{
e5=e4;
e4=e3;
e3=e2;
e2=e1;
e1=e;
e=d;
d=c;
c=b;
b=a;
a = analogRead(sensB);
}
avg=(a+b+c+d+e+e1+e2+e3+e4+e5)/10;
//channel[0] = analogRead(sensA);
channel[1] = analogRead(sensB);
Serial.print(moPos);
Serial.print("\t");
Serial.print(moPos+(iter*.1*direc));
Serial.print("\t");
Serial.println(avg);
iter = iter + 1;
delay(100);
}

```
```

        for(int x = 0; x < steps; x++)
        {
            digitalWrite(puls, HIGH);
            delay(1);
            digitalWrite(puls, LOW); // emits a 2ms pulse, advancing 1 step
            delay(1);
        }
        moPos = moPos - steps;
    }
    exit(0);
    }

```
```

